

A friend asked me what is the neutrino index of refraction and if it could be measured using the entire planet. The number is known to be very small, but I could not find the numerical answer anywhere.

I spoke to Bill Marciano who had written a preprint with the formula.

The index is almost always used in the context of oscillations. And small difference in the indices leads to big effects.

Numerically it is amazing that matter effect oscillations are sensitive to such small values of index of refraction.

Neutrino Index of refraction calculation  
using Marciano/Parsa hep-ph/0403168

In this note, I have calculated the actual index of refraction of the neutrino for various matter types.

**Setup some constants.**

```
In[1]:= inu = +1; (* neutrino or antineutrinos *)
In[2]:= nutype = "e"; (* neutrino flavor *)
In[3]:= pnu = 1; (* neutrino momentum in GeV *)
In[4]:= hbarcwunit = (UnitConvert[Quantity["PlanckConstant"], "GeV*Sec"] / (2 Pi)) *
          UnitConvert[Quantity["SpeedOfLight"], "cm/sec"]
Out[4]= 1.9732698 × 10-14 cm GeV

In[5]:= hbarc = QuantityMagnitude[hbarc];
gfermi = UnitConvert[Quantity["FermiCouplingConstant"], "Gev^-2"]
Out[5]= 0.0000116638/GeV2

In[6]:= gfermi * hbarcwunit^3
Out[6]= 8.9619 × 10-47 cm3GeV
```

**Setup the matter density in e, and u and d quarks.**

```
In[7]:= den = 1; (* matter density in gm/cm^3 *)
pfraction = 0.5; (* proton fraction This would be *)
nfraction = (1 - pfraction); (* neutron fraction *)
navo = UnitConvert[Quantity["AvogadroNumber"], 1];
pdensity = den * pfraction * navo;
edensity = den * pfraction * navo;
ndensity = den * nfraction * navo;
udensity = 2 * pdensity + ndensity; (* density of u quarks *)
ddensity = pdensity + 2 * ndensity; (* density of d quarks *)
```

## Weak couplings.

```
In[®]:= T3up = 1/2; (* weak isospin *)
T3dn = -1/2;
T3e = -1/2;
Sin2thw = 0.223; (* weak angle *)
Qup = 2/3; (* quark charges *)
Qdn = -1/3 ;
Qe = -1;

In[®]:= nindex[typ_] := Module[{tempe, tempu, tempd, const, temp},
  const = QuantityMagnitude[gfermi * hbarcunit^3];
  If[typ == "s", Return[0]]; (* include sterile neutrino *)
  If[typ != "e",
    tempe = -inu * Sqrt[2] * const * edensity * (T3e - 2 * Qe * Sin2thw) / pnu,
    tempu = -inu * Sqrt[2] * const * edensity * (1 + T3e - 2 * Qe * Sin2thw) / pnu];
  tempu = -inu * Sqrt[2] * const * udensity * (T3up - 2 * Qup * Sin2thw) / pnu;
  tempd = -inu * Sqrt[2] * const * ddensity * (T3dn - 2 * Qdn * Sin2thw) / pnu;
  temp = tempe + tempu + tempd; Return[temp]
]

In[®]:= nindex["e"]
Out[®]= -1.90812 × 10-23
```

Do some checks. First difference between e and mu type

```
In[®]:= nindex["e"] - nindex["mu"]
Out[®]= -3.81623 × 10-23
```

Calculate the potential  $\text{Sqrt}[2]*\text{Gf}*\text{pfraction}*(\text{Nf}/6*10^{23}/\text{cc})$  in eV.

```
In[®]:= Sqrt[2] * gfermi * hbarcunit^3 * navo *
Quantity[1, "cm^-3"] * Quantity[10^9, "eV/GeV"] * pfraction
Out[®]= 3.81623 × 10-14 eV
```

Let's calculate this for various types of matter.  
first ordinary matter in Earth with density of ~5 gm/cc or rock.

```
In[8]:= den = 5; (* matter density in gm/cm^3 *)
pfraction = 0.45; (* proton fraction This would be *)
nfraction = (1 - pfraction); (* neutron fraction *)
navo = UnitConvert[Quantity["AvogadroNumber"], 1];
pdensity = den * pfraction * navo;
edensity = den * pfraction * navo;
ndensity = den * nfraction * navo;
udensity = 2 * pdensity + ndensity; (* density of u quarks *)
ddensity = pdensity + 2 * ndensity; (* density of d quarks *)
Print[" electron ", nindex["e"], " mu and tau ", nindex["mu"]]
electron -6.67841×10-23 mu and tau 1.04946×10-22
```

## Star matter inside the Sun core

```
In[9]:= den = 150; (* matter density in gm/cm^3 *)
pfraction = (1 * 0.33 + 0.5 * 0.67);
(* proton fraction This would be let's use 33% hydrogen and 67 % He *)
nfraction = (1 - pfraction); (* neutron fraction *)
navo = UnitConvert[Quantity["AvogadroNumber"], 1];
pdensity = den * pfraction * navo;
edensity = den * pfraction * navo;
ndensity = den * nfraction * navo;
udensity = 2 * pdensity + ndensity; (* density of u quarks *)
ddensity = pdensity + 2 * ndensity; (* density of d quarks *)
Print[" electron ", nindex["e"], " mu and tau ", nindex["mu"]]
electron -5.69573×10-21 mu and tau 1.91766×10-21
```

white dwarf matter (density ranges from  $2 \times 10^9 \text{ kg/m}^3$  to  $10^{11} \text{ kg/m}^3$  )

```

den = 2 * 10^6; (* matter density in gm/cm^3 *)
pfraction = 0.5; (* proton fraction this would be *)
nfraction = (1 - pfraction); (* neutron fraction *)
navo = UnitConvert[Quantity["AvogadroNumber"], 1];
pdensity = den * pfraction * navo;
edensity = den * pfraction * navo;
ndensity = den * nfraction * navo;
udensity = 2 * pdensity + ndensity; (* density of u quarks *)
ddensity = pdensity + 2 * ndensity; (* density of d quarks *)
Print[" electron ", nindex["e"], " mu and tau ", nindex["mu"]]
electron  $-3.81623 \times 10^{-17}$  mu and tau  $3.81623 \times 10^{-17}$ 

```

neutron star matter (density of  $2 \times 10^{17} \text{ kg/m}^3$ )

```

In[]:= den = 2 * 10^14; (* matter density in gm/cm^3 *)
pfraction = 0.005; (* proton fraction this is needed for stability *)
nfraction = (1 - pfraction); (* neutron fraction *)
navo = UnitConvert[Quantity["AvogadroNumber"], 1];
pdensity = den * pfraction * navo;
edensity = den * pfraction * navo;
ndensity = den * nfraction * navo;
udensity = 2 * pdensity + ndensity; (* density of u quarks *)
ddensity = pdensity + 2 * ndensity; (* density of d quarks *)
Print[" electron ", nindex["e"], " mu and tau ",
      nindex["mu"], " diff ", nindex["e"] - nindex["mu"]]
electron  $7.51798 \times 10^{-9}$  mu and tau  $7.5943 \times 10^{-9}$  diff  $-7.63247 \times 10^{-11}$ 

```